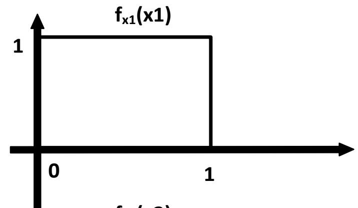
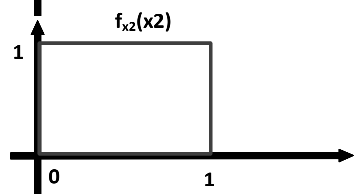
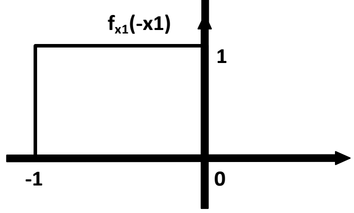
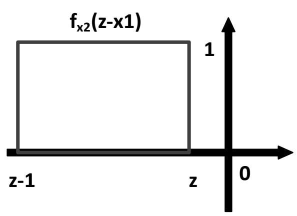








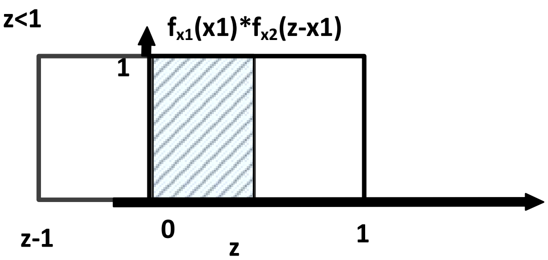
$$f_z(z) = \int_{-\infty}^{\infty} f_{x1}(x1) f_{x2}(z - x2)dx$$

$$z = X1 + X2$$

$$0 \le z < 1$$

$$f_z(z) = \int_0^z 1dx = x|_0^z = z$$

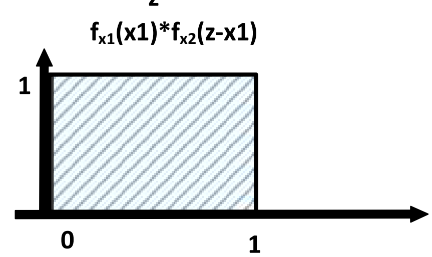
0≤ z<1



$$z = 1$$

$$f_z(z) = \int_0^1 1dx = x|_0^1 = 1$$

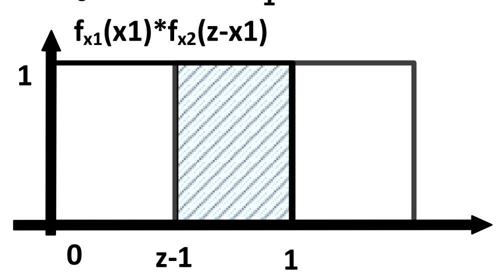
z=1



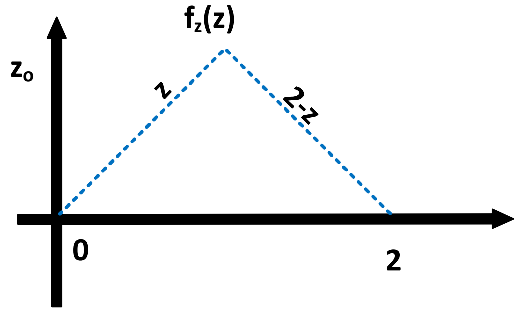
$$1 < z \le 2$$

$$f_z(z) = \int_{z-1}^1 1dx = x|_{z-1}^1 = 1 - z + 1 = 2 - z$$

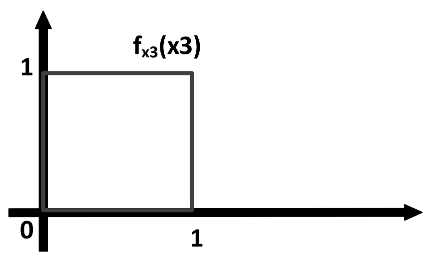
1< z<2

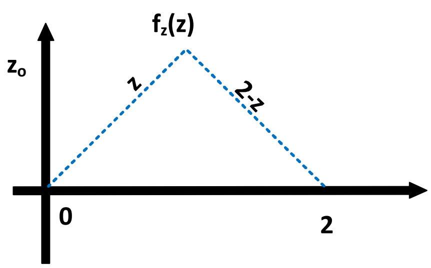


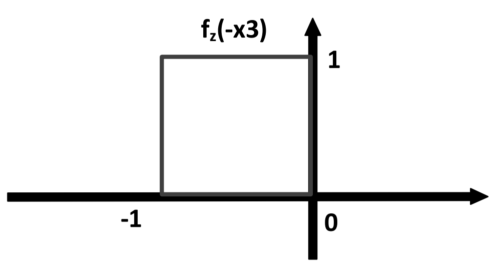
$$f_z(z) = \begin{cases} z & 0 < z < 1 \\ 2 - z & 1 < z < 2 \\ 0 & otherwize \end{cases}$$

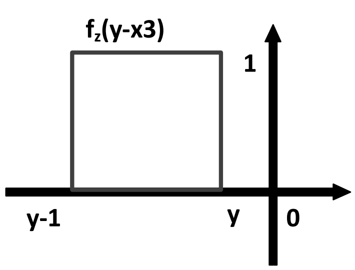


$y = z + X3$



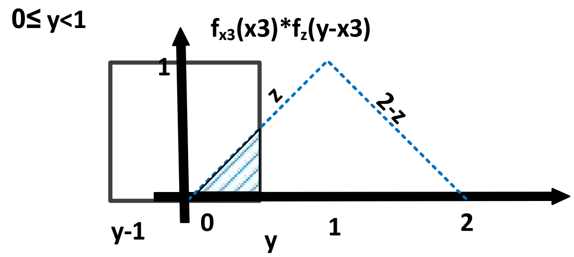


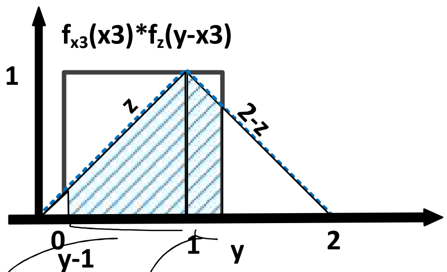




$0 \le y < 1$

$$f_y(y) = \int_0^y z\,dz = \frac{z^2}{2}\Big|_0^y = \frac{y^2}{2}$$





$1 \le y < 2$

$$f_y(y) = \int_{y-1}^{1} z\,dz + \int_1^y (2-z)\,dz = \frac{z^2}{2}\Big|_{y-1}^1 + 2z - \frac{z^2}{2}\Big|_1^y = \frac{1^2}{2} - \frac{(y-1)^2}{2} + 2y - \frac{y^2}{2} - 2*1 + \frac{1^2}{2}$$

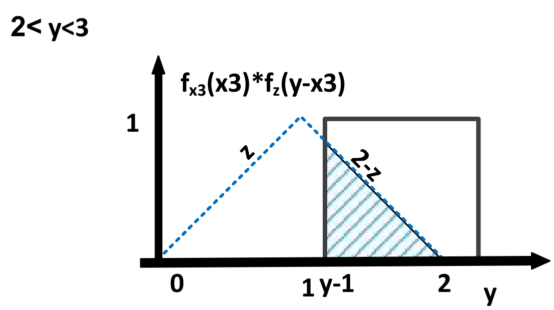
$$= -\frac{y^2 - 2y + 1}{2} - 1 + 2y - \frac{y^2}{2} = -y^2 + 3y - \frac{3}{2}$$

$2 \le y < 3$

$$f_y(y) = \int_{y-1}^{2} (2-z)\,dz = 2z - \frac{z^2}{2}\Big|_{y-1}^2$$

$$= 2(2) - \frac{2^2}{2} - 2(y-1) + \frac{(y-1)^2}{2}$$

$$= \frac{y^2 - 2y + 1}{2} - 2(y-1) + 2 = \frac{y^2}{2} - 3y + \frac{9}{2}$$



$$f_y(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y^2}{2} & 0 \le y < 1 \\ -y^2 + 3y - \dfrac{3}{2} & 1 \le y < 2 \\ \dfrac{y^2}{2} - 3y + \dfrac{9}{2} & 2 \le y < 3 \\ 0 & 3 \le y \end{cases}$$